SOLVING RATIONAL INEQUALITIES

Objectives:
1) Solve rational inequalities algebraically (using a sign chart)
2) Solve rational inequalities using a base graph

In order to solve rational inequalities it is necessary to find both the zeros and where the inequality is undefined.

Example 1: Solve \( \frac{6}{x} + 2 \geq 0 \).

Simplify the left side by finding a common denominator. \( \frac{6 + 2x}{x} \geq 0 \).

The zero from the numerator is \( x = -3 \) and inequality is undefined at \( x = 0 \). These are called critical numbers.

Place the critical numbers -3 and 0 on the number line

Arbitrarily choose a value in each interval and determine whether + or -.

The solution includes the only one of the critical numbers. The value \( x = 0 \) cannot be part of the solution because the inequality is not defined there. Hence the solution is the intervals \( (-\infty, -3] \) and \( (0, \infty) \).

Example 2: Solve \( \frac{4}{x - 3} < 2 \).

First step is to set the inequality to zero. \( \frac{4}{x - 3} - 2 < 0 \).

Simplify the left side by finding a common denominator. \( \frac{10 - 2x}{x - 3} < 0 \).

The zero from the numerator is \( x = 5 \) and the inequality is undefined at \( x = 3 \).

Place the critical numbers 5 and 3 on the number line
Arbitrarily choose a value in each interval and determine whether + or -.

The solution includes none of the critical numbers since there is not equal sign. Hence the solution is the intervals \((-\infty, 3)\) and \((5, \infty)\).

SOLVE USING THE BASE GRAPH.

Example 3: Solve \(\frac{6}{x} + 2 \geq 0\).

Graph the base graph \(y = \frac{1}{x}\) with a horizontal translation of 2 units. See the graph at the right.

For what interval(s) along the x-axis is the graph above zero, meaning above the x-axis?

Answer: \((-\infty, -3]\) and \((0, \infty)\).

Example 4: Solve \(\frac{4}{x - 3} < 2\).

Graph the base graph \(y = \frac{1}{x}\) with a vertical translation of 3 units to the right. See the graph at the right.

For what interval(s) along the x-axis is the graph below the line \(y = 2\).

Answer: \((-\infty, 3)\) and \((5, \infty)\).
Rational Inequality Handout

Solve each of the following graphically. Write your answer in interval notation.

1. \( \frac{1}{x} + 1 \geq 0 \)

2. \( \frac{1}{x^2} - 1 \leq 0 \)

3. \( \frac{3}{x + 1} \leq 3 \)

4. \( \frac{1}{(x - 2)^2} \leq 1 \)

Solve each of the following algebraically. Write your answer in interval notation.

5. \( \frac{6}{x} < 2 \)

6. \( \frac{x + 3}{x - 1} \geq 0 \)

7. \( \frac{x(4 - x)}{x + 2} \geq 0 \)

8. \( \frac{(x + 3)^2}{x} \leq 0 \)

9. \( \frac{x^2 - 4}{3 - x} \geq 0 \)

10. \( \frac{-5}{(x + 3)^2} > 0 \)

11. \( \frac{2x}{x - 2} \leq 3 \)

12. \( \frac{1}{x + 2} \geq \frac{1}{3} \)

13. \( \frac{1}{4} < \frac{7}{7 - x} \)

14. \( \frac{x + 2}{x + 5} \geq 1 \)

15. \( \frac{3}{x - 2} \leq \frac{3}{x + 3} \)

16. \( x - \frac{10}{x - 1} \geq 4 \)

17. What is the domain of each of the following functions.

a) \( y = \sqrt{x^2 - 4} \)

b) \( y = \sqrt{\frac{x + 1}{x - 2}} \)